CHAPTER 6
CONSUMER DEMAND

One goal of this course is to instill an understanding of how markets work, which includes how prices are formed and what makes prices change. Prices refer to a trade between people, and in every trade there is a buyer and a seller. Rather than take on the behavior of both buyers and sellers at the same time, we study them separately, and then see what happens when we put them together. This is analogous to learning how a car works by first learning how the pistons, radiator, water pump, and other parts work, rather than taking on the entire car at one time. For ever trade, there is a buyer and a seller. In this chapter, we will study how buyers (consumers) behave.

6.1) The Demand Curve

Consumer behavior, at least in this class, is described by demand curves. A demand curve is a graph for one particular good. It is a “picture” of how consumers react under different circumstances. It describes how many units of the good people will buy at different prices during a specific period of time. For this reason, it is often referred to as a demand schedule, a schedule of prices and quantities.

Consumer demand has a particular set of unique terms, and economists can be quite picky on using the correct terminology. A demand curve can be drawn for almost any good. It can be a highly differentiated product, like Samual Adams Octoberfest beer, or a general product like “beer.” Similarly, the demand curve can be for T-bone steaks, or beef in general. Demand curves can even be drawn for a generic good “food.” Or for those studying the whole economy (macroeconomists), there is a demand curve for “all goods and services.”

First, let us discuss what is on each axis. The real price of the good is on the y-axis. We use real prices, as opposed to nominal prices, because it is real prices that determine how much consumers purchase. The units for “quantity” can be almost anything, but you want to make sure you are adding identical units, or almost identical units. For instance, if we are talking about the demand for beef, the quantity might be lbs of beef consumed. If meat, we can use a measure of total lbs of pork, beef, chicken, and poultry consumed. However, for computer demand, you would not want to add together total computers purchased if some are very old computers and some are fast, new computers. Instead, you may want to use something like “megahertz” as a quantity of computer power. Some may argue, then, that we shouldn’t add together beef from 20 years ago to beef today, because beef has changed. I agree partially, but

Demand Curve: A graph with the real price on the y-axis and the quantity consumers purchase on the x-axis. At any price, this curve indicates how many units consumers will purchase at that price during a specific period of time.

THE DEMAND CURVE FOR BEEF

\[ D = \text{Demand} \]

\[ P = \text{real price / lb beef} \]

\[ Q = \text{lbs meat purchased / month} \]
many contend beef has not changed enough to make this a significant problem.

Also, you want to make sure that the real price (hereafter, price) used matches the quantity. For instance, you would not want to use the price of all beef meat (which contains lower quality cuts of meat) for the quantity of rib eyes purchased. Instead, you would use the price of rib eyes for the quantity of rib eyes purchased.

The quantity variable must be relevant to some time period, like quantity purchased per week or year.

Economists generally make few assumptions regarding how the demand curve looks. We do require it to possess one trait though: IT MUST SLOPE DOWNWARD. This is referred to as the First Law of Demand. No matter what good we look at, when the price goes up, and if everything else in the world is held constant, quantity purchased goes down. Economists have even tested the First Law of Demand on animals, like hamsters.¹

The demand curve tells us how much quantity demanded will fall in response to an increase in price (or conversely, how much quantity demanded will rise in response to an decrease in price) holding all else in the universe constant (ceteris paribus).²

Using statistical techniques, we can estimate demand curves from data. Below is a plot of lbs of beef consumed per month and the price of beef each month in real 1990 dollars. “Beef” here is defined as boxed beef, which is basically a carcass (without the skin or guts). The dots show the quantity demanded at various prices between 1986 though 1999. It may be difficult see the First Law of Demand in the graph. Sometimes, in the data, both prices and quantity rose at the same time, but that was likely due to factors other than price. For instance, if prices and incomes rise at the same time, quantity may increase due to the rise in income, and it will look like higher prices caused higher quantities. However, when we use statistical techniques to estimate the impact of price on quantity demanded,³ I get a downward sloping demand curve as shown below.

Now, get ready for some picky technology. We call the quantity purchased the quantity demanded. If the [real] price increases, causing consumers to buy less, we say there has been a decrease in quantity demanded. Similarly, if price rises, we say there has been a movement along the demand curve, not a change in demand.

¹ The hamster was allowed to receive food by pressing a lever. Economists then increased the price of food by making the lever harder to press. As it took more strength to press the lever, hamsters consumed less food. That is, as price went up, quantity went down.
² Ceteris paribus is Latin for “all things being equal” which is better translated as “all else held constant.”
³ These statistical techniques are able to "hold all else constant," and thus enable you to determine the effect of price and price alone on quantity demanded.
Simply put, demand refers to how the demand curve looks, while quantity demanded refers to how much consumers purchase at a particular price. Quantity demanded is one point on the demand curve.

**Nit-Picky Terminology**

A change in price causes a change in quantity demanded (not a change in demand). A change in price causes movement along the demand curve (not a change in the demand curve).

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**FIGURE 6.A**

**THE DEMAND CURVE FOR BEEF**

Note: These are real monthly beef data for US consumers from 1986 through 1999.

6.2) **Determinants of Demand**

Economists know that price is not the only thing that influences consumer purchases. The price of other goods, income, perceptions, and many other variables also matter. These other variables may cause the demand curve to “shift.” For instance, at a particular price for fried chicken I will buy a certain amount. However, come football time when my weekends are spent tailgating, I will buy more chicken at that same price.

Economists say the variables that determine how much of a good consumers purchase are:

**Determinants of Demand**

1) [real] Price of the good
2) [real] Price of other goods
3) [real] Income
4) Number of buyers
5) Tastes and Preferences
6) Expectations
When the own-price of the good (Determinant 1) changes, we say that there has been a *change in quantity demanded* or a *movement along the demand curve*. But when Determinants 2-6 change, we say there has been a *change in demand* or a *shift in the demand curve*. Consider the following examples below for how each determinant influences consumer demand. In each example, I will tell you what changes and then show you how that changes demand. Unless I state otherwise, everything else remains the same (e.g. if I tell you income changes, assume tastes and preferences remain the same).

**Example 1: Change in Own Price**

The price of Slim Jims falls from $P$ to $P'$. What will happen to the demand for Slim Jims?

*There will be a movement along the demand curve. The quantity demanded increases from $Q$ to $Q'$.*

**Example 2: Change in Price of Other Goods**

Fried Chicken and tailgating are complements (meaning they go together; people like consuming them together and if one is cheaper they will by more of the other). What will happen to the demand for Fried Chicken if the price of tailgating goes down (like the price of parking for tailgating)?

*People will tailgate more, and thus will consume more fried chicken at any price. The demand for Fried Chicken increases. At the price of $P$, people will now buy $Q'$ which is more than they bought before the demand shift ($Q$). The demand curve shifts from $D$ to $D'$ (it shifts up or to the right, however you see it).*

**Example 3: Change in Price of Other Goods**

For many people, beer and wine are substitutes, meaning when the price of one gets very high they will substitute it with the other. What happens to the demand for wine if the price of beer falls?

*The demand for wine will decrease as consumers substitute beer for wine. The demand curve shifts down (or left). At price $P$, consumers will decrease consumption from $Q$ to $Q'$.*
Example 4: Change in Price of Other Goods

Consider the market for types of beer, within the general beer market. Are Samuel Adams Light and Keystone Light substitutes or complements? What happens to the demand for Samuel Adams Light if the price of Keystone Light falls?

*It would seem that Samuel Adams Light and Keystone Light are substitutes. Thus, if the price of Keystone Light falls, the demand for Samuel Adams Light will fall. At Price $P$, quantity consumed will decrease (from $Q$ to $Q'$), and the demand curve shifts downward (or to the left). The demand for Samuel Adams Light decreased.*

Example 5: Change in Income

An inferior good is one in which people consume less as their incomes rise. Ramen noodles are an example. College students eat tons of it, but when they graduate and get jobs, they rarely eat it again. Illustrate what happens to the demand for an inferior good if people’s income falls.

*If incomes fall, the demand for inferior goods will rise. The demand curve for inferior goods will shift up (or to the right). The demand curve shifts from $D$ to $D'$. At price $P$, consumption rises from $Q$ to $Q'$.***

Example 6: Change in Income

A normal good is one in which people consume more as their incomes rise. Food, like most goods, are normal goods. Show in the corresponding graph what happens to food demand as a nation experiences a recession and incomes fall.

*The demand for food falls in response to a decrease in income because it is a normal good. The demand curve shifts down (or to the left) and at price $P$ consumption will fall from $Q$ to $Q'$.***
Example 8: Change in Tastes and Preferences

Texas put Oprah Winfrey in jail for talking bad about beef. It is likely Texas will execute her. What happened to the demand for beef when Oprah claimed it was high in cholesterol and thus unhealthy to eat?

The demand for beef decreased. The demand curve shifted from D to D’. Quantity consumed at price P decreased from Q to Q’.

Example 9: Change in Expectations

New studies find that the increase in expected salary from getting a college education is $10,000 more than previously thought. How will the demand for college education be impacted?

Due to a change in expectations, the demand for college education will increase.

Example 9: Change in Number of Buyers

A group of Southeastern Oklahoma cattlemen are considering opening a cooperative in Southeastern Oklahoma which will slaughter bulls and cull cattle (old momma cows that are now long productive; meat from bulls and cull cattle are low quality meat). They will be buying their cattle from Oklahoma, North Texas, Western Arkansas and Western Louisiana. After the cooperative opens, what will happen to the demand for bulls and cull cattle for slaughter in Oklahoma?

Since there are now more buyers, the demand for bulls and cull cattle for slaughter will increase. The demand curve will shift from D to D’, and at price P, buyers will increase demand from Q to Q’.
6.3) Analyzing Consumer Demand

The previous examples were hypothetical changes in demand. Sometimes, we are interested in taking data and looking at what really happens to demand when the determinants of demand change. For instance, beef has lost market shares to pork and poultry. Since pork and poultry and substitutes for beef, the fact that pork and poultry prices have fallen and several health concerns has caused people to substitute beef for pork and/or poultry. Suppose we want to know pork or chicken is a stronger substitute for beef. We can make guesses, or look at consumer data and estimate which has a stronger substitution effect. In this section, I will walk you through an example where we will take real data and analyze beef demand to answer this question. First, we have to decide exactly what this “beef” good is. What quantity units should we use?

6.3.A) Choose good of interest

I will choose to use the total lbs of boxed beef purchased per month. Boxed beef is basically the carcass (the cow without the head, hide, guts, …, just the meat and bones). This is not the beef you buy in the grocery store; it is wholesale beef. Therefore, the price I will use is the per pound price of boxed beef, which is a wholesale price. After inspection, I see that my price and quantity units are compatible.

6.3.B) Collect data on the good and its demand determinants

Then, I run down the list of demand determinants in the previous section, and collect data on those variables.

*Determinants of monthly lbs of boxed beef demand*

I first collect data on the total lbs of boxed beef consumed each month from February, 1986 through December, 1999. Then, I obtain data on other variables that influence boxed beef demand for these same months. These data are as follows.

1) Price of the good (Obtain per cwt (100 pounds) price of boxed beef data)
2) Price of other goods (I decide that the goods most related to beef are pork and poultry, because they are substitutes for meat. Thus, I collect data on the per cwt prices of wholesale pork and chicken meat. The wholesale pork and chicken variables are the lbs of pork and chicken carcass slaughtered. While I acknowledge that French fries may be a complement for beef, and therefore is another demand determinant, I do not believe the complementary is stronger enough to warrant me collecting French fry price data. We could go on and on naming other relevant goods, but at some point you must draw the line and identify the most relevant good for inclusion in the demand analysis. Every analysis is incomplete to some degree.)
3) Income (Collect data on US disposable income; which is income for the country after taxes)
4) Number of buyers (Collect data on the total population each month from 1986 through 1999)
5) Tastes and Preferences (It is difficult to find data that reasonable measures “tastes and preferences” so I ignore this determinant for now)
6) Expectations (It is difficult to find reasonably measures of “expectations” so I ignore this determinant)
6.3.C) Check data, make sure all prices and variables with money units are in compatible real prices

Four variables have money as their units. Boxed beef, pork, and poultry prices are nominal prices, and disposable income is nominal income. We want to convert these nominal variables to real variables, i.e., we want to change all nominal dollars into real dollars.

To accomplish this, I obtain monthly data on the Consumer Price Index (CPI) for 1986 through 1999. The base year for this CPI is 1984. I convert this CPI so that its base year is 1990 by multiplying each CPI value by \((100 / \text{average monthly CPI value in 1990})\). See Chapter 4 if this is unclear. Then, I use the CPI with a 1990 base year to convert all nominal prices and nominal disposable income to real prices and real disposable income in 1990 dollars. This is exactly what we did in Homework #1. Again, see Chapter 4 if this is unclear.

What we have now are data that can be used to study the demand for boxed beef. We have one variable we are tying to explain, and five variables that are doing the explaining.

*Variable To Explain:*

Demand for Boxed Beef, defined as the total lbs of boxed beef purchased each month in the US.

*Variables Doing the Explaining (CWT = 100 Pounds):*

1) Real Per CWT Price of Boxed Beef in 1990 Dollars
2) Real Per CWT Price of Wholesale Pork in 1990 Dollars
3) Real Per CWT Price of Wholesale Chicken in 1990 Dollars
4) Real US Disposable Income in Billion 1990 Dollars
5) US Population in Thousands

6.3.D) Estimate an equation that best predicts beef demand (lbs boxed beef consumed per month) as a function of the demand determinants; the own-real-price, the real price of pork, the real price of chicken, real disposable income, and population.

This is accomplished as follows. I arrange the data in a spreadsheet as shown below.
The data are in rows 6 through 172.

I then use an excel function to estimate a demand equation. This is accomplished as follows. Choose “Tools” then “Data Analysis” then “Regression.” Your screen will look like this:
You can see the data are in columns C, D, E, F, G, and H.

The data begin in row 6 and end in row 172.

Notice I created data labels in row 5, as you will see later, this will make things easier to understand.

Now, you must tell Excel which data contains the variable you are trying to explain (beef demand) and which data are doing the explaining (all other variables). For Excel, the variable being explained is the “Y” variable, so you must tell Excel your Y variable (beef demand) is in cells C6:C172. Enter this in Excel as follows:
Next, you must tell Excel where the data doing the explaining (the determinants of demand) are located. Excel calls these the “X” variables and you can enter this information by doing the following:

Finally, you must tell Excel that you did include data labels and that you want it to show the results in a new worksheet.
When you press “OK”, excel will estimate a demand equation for you and put it in a new worksheet. The output will appear as follows.

The output contains a lot of statistics, none of which you are likely familiar with. We are only concerned with one set of numbers, the ones circled under “Coefficients.”

You can ignore all the other numbers.

These numbers under “coefficients” give us an equation that we can use to predict beef demand. Notice each number is associated with a particular data label. These data label / number combinations can be used to construct a demand equation as follows.
Estimated Demand Equation:

Quantity Demanded of Boxed Beef in Million Pounds = (86) + (-0.2545)(P Beef) + (0.0777)(P Pork) + (0.1731)(P Chicken) + (0.0123)(Income) + (-0.0002)(Population)

or (recall CWT = 100 Pounds)

\[
\begin{align*}
86 & \\
+ (-0.2545) & \text{(Real Per CWT Price of Boxed Beef in 1990 Prices)} \\
+ (0.0777) & \text{(Real Per CWT Price of Wholesale Pork in 1990 Prices)} \\
+ (0.1731) & \text{(Real Per CWT Price of Wholesale Chicken in 1990 Prices)} \\
+ (0.0123) & \text{(Real Disposable Income in 1990 Prices)} \\
+ (-0.0002) & \text{(US Population in Thousands)}
\end{align*}
\]

= Quantity Demanded of Boxed Beef in Million Pounds Per Month

To give you an example of how we can use this equation, I will insert the average of each variable in the equation to predict average beef demand.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Variable Units</th>
<th>Average Variable Value From 1986-1999</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price of Boxed Beef</td>
<td>101</td>
<td></td>
</tr>
<tr>
<td>Price of Wholesale Pork</td>
<td>Real Per CWT Prices in 1990 Dollars</td>
<td>101</td>
</tr>
<tr>
<td>Price of Wholesale Chicken</td>
<td>51</td>
<td></td>
</tr>
<tr>
<td>Disposable Income</td>
<td>Real 1990 Dollars in Billions</td>
<td>4,489</td>
</tr>
<tr>
<td>US Population</td>
<td>Thousands</td>
<td>258,861</td>
</tr>
</tbody>
</table>

Demand Equation For Boxed Beef:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Variable Value</th>
<th>Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price of Beef</td>
<td>101 *</td>
<td>-0.2545</td>
</tr>
<tr>
<td>Price of Pork</td>
<td>+ 101 *</td>
<td>0.0777</td>
</tr>
<tr>
<td>Price of Chicken</td>
<td>+ 51 *</td>
<td>0.1731</td>
</tr>
<tr>
<td>Income</td>
<td>+ 4,489 *</td>
<td>0.0123</td>
</tr>
<tr>
<td>Population</td>
<td>+ 258,861 *</td>
<td>-0.0002</td>
</tr>
</tbody>
</table>

Million Pounds Boxed Beef Per Month = 77.1694
The “coefficients” give the demand curve its shape and tells how they shift when other variables change. The variables are data being used to explain beef demand. This equation predicts that from 1986 through 1999, the average beef demand was 77 million lbs. It so happens that this equation is accurate—the average beef demand was 77 million lbs. By plugging in other numbers for the variable values, you can see how the demand curve changes in response to conditions. This tells you how consumer behavior will differ when the marketplace changes.

6.3.E) Learn From Your Demand Equation

This equation provides an insightful tool for making statements about beef demand. It can be used to answer the following questions.

**Question: Does beef demand exhibit the First Law of Demand?**

This question is asking, when the price of beef rises (and we are always talking about real prices when it comes to demand), does the quantity demanded decrease? We can answer this from the equation above. The coefficient on the price of boxed beef is -0.2545. This means that if the price of boxed beef increases by one, quantity demanded falls by -0.2545. For instance, if we change the equation above to let the own price increase by one, we predict the quantity demanded will be

```
Demand Equation For Boxed Beef:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price of Beef</td>
<td>101+1 = 102</td>
<td>-0.2545</td>
</tr>
<tr>
<td>Price of Pork</td>
<td>+ 101</td>
<td>0.0777</td>
</tr>
<tr>
<td>Price of Chicken</td>
<td>+ 51</td>
<td>0.1731</td>
</tr>
<tr>
<td>Income</td>
<td>+ 4,489</td>
<td>0.0123</td>
</tr>
<tr>
<td>Population</td>
<td>+ 258,861</td>
<td>-0.0002</td>
</tr>
</tbody>
</table>

Million Pounds Boxed Beef = 76.9149
```

Average Quantity Demanded of Boxed Beef in Million Pounds = (86) + (-0.2545)(101+1 = 102) + (0.0777)(101) + (0.1731)(51) + (0.0123)(4,489) + (-0.0002)(258,861) = 76.9149 million lbs.

This is lower than the quantity demanded of 77.1694 when the beef price was 101.

When price goes up, quantity demanded does indeed go down. We could plug in many different prices and observe the corresponding change in quantity demanded. We could also plot these points on a graph to get a demand curve. When I hold all the other variables at their average values as shown above, and graph the relationship between beef demand and beef prices, I get the following demand curve.
As you see, this demand curve does exhibit the First Law of Demand.

*Question: Are beef and pork substitutes or complements?*

As explained previously in this chapter, complements are things that “go together”, meaning people like to consume them together. Substitutes, as its name implies, means consumers can substitute one for another. If an increase in the price of pork causes beef demand to increase, then they are substitutes. Consumers are responding to higher pork prices by substituting pork with beef. On the other hand, if an increase in pork prices causes a decrease in beef demand, they are complements.

Let us use our demand equation to see how beef demand responds to higher pork prices. Recall the equation is

\[
Q = 86 - 0.2545(\text{Real Per CWT Price of Boxed Beef in 1990 Prices}) \\
+ 0.0777(\text{Real Per CWT Price of Wholesale Pork in 1990 Prices}) \\
+ 0.1731(\text{Real Per CWT Price of Wholesale Chicken in 1990 Prices}) \\
+ 0.0123(\text{Real Disposable Income in 1990 Prices}) \\
- 0.0002(\text{US Population in Thousands})
\]

= Quantity Demanded of Boxed Beef in Million Pounds Per Month

We can tell from the equation that when the price of pork rises, the demand for beef rises. To see how the demand curve shifts, below are the demand curves when the price of pork is 100 and 200.
Since consumers respond to higher pork prices by consuming more beef, we would answer than beef and pork are substitutes.

**Question: Is Beef a Normal or Inferior Good?**

Our demand equation can answer this. When we analyze the demand equation

(86)

\[
\begin{align*}
&\quad + (-0.2545)(\text{Real Per CWT Price of Boxed Beef in 1990 Prices}) \\
&\quad + (0.0777)(\text{Real Per CWT Price of Wholesale Pork in 1990 Prices}) \\
&\quad + (0.1731)(\text{Real Per CWT Price of Wholesale Chicken in 1990 Prices}) \\
&\quad + (0.0123)(\text{Real Disposable Income in 1990 Prices}) \\
&\quad + (-0.0002)(\text{US Population in Thousands}) \\
\end{align*}
\]


= Quantity Demanded of Boxed Beef in Million Pounds Per Month

we see that the coefficient on real income is 0.0123, a positive number. This means an increase in the US income will increase the demand for beef. The demand curve will shift up and to the left as in the previous example. Beef is a normal good.

**Question: Which is a Stronger Substitute For Beef; Pork or Chicken?**

The beef industry may want to know this to use in their advertising campaigns. Many campaigns are “anti” other products, so is beef competing more against pork or chicken? Our demand equation above suggests that a one dollar decrease in the price of chicken lowers beef demand by 0.1731 million pounds per month, while a one dollar decrease in the price of pork lowers beef demand by 0.0777 million pounds.

Thus, we conclude that chicken is a stronger substitute for beef than pork.
Suppose a tax is placed on the consumption of alcohol. By how much will consumers be impacted and what tax revenues will be raised? If the cost of crop production increases due to environmental regulation, by how much can those costs be passed on to the consumer? The answer to these and many other economic questions depends on the elasticity of demand.

There are several different types of demand elasticities, but the most frequently used is the own-price elasticity of demand. The own-price elasticity of demand is a measure of how sensitive quantity demanded is to a change in price. It simply measures how sensitive consumers are to price changes.

Types of Demand Elasticities

1. own-price elasticity of demand
2. cross-price elasticity of demand
3. income elasticity of demand

Suppose we wish to compare how sensitive consumers are to changes in the price of cars in a rural versus an urban area. Two demand curves for each area are shown above. People in urban areas have more choices of transportation than rural folks. Urban folk can take subways or buses instead of cars, so if the price of cars rises, many will use these other transportation modes rather than a car, and the quantity demanded of cars will decrease just as shown in the graph above. Rural people have fewer choices regarding transportation though, must rely exclusively on cars. Therefore, an increase in the price of cars will not cause the quantity demand of cars to decrease as much in the rural area as it did in the urban area. *The more substitutes a good has, the more sensitive consumers will be to changes in the price of that good.*

For this example, we could simply measure “sensitivity to price” as the decrease in number of cars purchased. We cannot always use “change in quantity demanded” as a measure of sensitivity though. Suppose the price of cars and candy bars increased by one dollar, and the quantity demanded decreased in both markets. How can compare “sensitivity to price” here? We cannot simply compare the increase in quantity demanded between cars and candy bars—we cannot compare cars to candy bars because they are different goods. It would be like comparing apples to oranges. Plus, it certainly seems that a one dollar increase in the price of cars is not comparable to a one dollar increase in the price of candy bars. So instead, economists developed a measure referred to as the own-price elasticity of demand. Instead of comparing price changes to quantity changes, elasticities compare *percent* changes in price and *percent* changes in quantities.
Refer back to when we estimated the demand for boxed beef. From our estimated demand equation, we were able to draw the demand curve shown to the right.

Using this curve, we can estimate the own-price elasticity of boxed beef demand. Suppose price fell from 120 to 100 $(1990 \text{ dollars}) / \text{cwt.}$ This is a $\frac{(100-120)}{120} = -0.17$ 17% decrease in price (the percent change is –17%).

Quantity demanded, as the graph shows, increased from about 72 to 77 million lbs / month. This is an increase in of $\frac{(77-72)}{72} = 0.07$ 7%. We would then estimate the own-price elasticity of demand as:

$\text{Own-Price Elasticity of Boxed Beef Demand} = \frac{\text{percent change in quantity demanded}}{\text{percent change in price}} = \frac{7\%}{-17\%} = -0.4.$

Elasticities have no units, they are simple ratios.
Applying own-price elasticities:

The US anticipates boxed beef imports to the US to increase by 10%. This means total boxed beef sold to consumers will increase by 10%. If the price of boxed beef is currently $90 / cwt in 1990 dollars, by how much will the price of boxed beef decrease to sell this extra 10%? Assume the own-price elasticity of demand is –0.4.

Answer:

The equation for own-price elasticity of demand (ED) is

\[ E_D = \frac{\% \Delta QD}{\% \Delta P} \]

Since we know \( E_D \) and what the percent change in quantity demanded must equal, we know the corresponding percent change in price.

\[-0.4 = \frac{(10\%)}{\% \Delta P} \]

\[% \Delta P = \frac{(10\%)}{-0.4} = -25\% \]

The 10% increase in imports to the US will cause a 25% decrease in boxed beef prices.

Notice that own-price elasticities will always be negative (due to the first law of demand). If prices rise, quantity demanded falls and vice-versa. In the own-price elasticity formula, either the numerator or denominator is negative, but not both.

A higher elasticity in absolute value means consumers are more sensitive to price changes. In the previous example quantity demanded increased from 72 to 77 million lbs. What if consumers had been more sensitive to price changes, and increased consumption up to 100 million lbs? The calculated elasticity would then be:

Own-price elasticity of demand = \( E_D = \frac{(100-72)/(72) = 0.39}{[(100-120)/(120) = -0.17]} = -2.29 \). This is much higher in absolute value than –0.4. Many times, instead of calculating elasticities, we try to visualize them. Consider the demand curve for food below.

I have drawn in a large price change, but according to the demand curve, quantity changes only a little (a big price drop increased quantity demanded by a little; a big price rise decreases quantity demanded by a little). This makes sense, because food is a necessity. Even if food prices rise dramatically, we still need food to live and so will curtail our consumption only a little. When big price changes cause little changes in quantity demanded, we say that demand is inelastic, which is another way of saying consumers are insensitive to price changes.

An extreme case of insensitivity to prices is insulin. Currently, there is no substitute for insulin; diabetics absolutely must have it to live, and they cannot alter the quantity of insulin they consume. Therefore, consumer purchases of insulin will be exactly the same regardless of prices (assuming they can still afford insulin), and the demand curve will appear as follows.

![Food Demand Curve](image-url)
This is a case of a perfectly inelastic demand. The elasticity actually equals zero, because regardless of the percent change in price, the percent change in quantity demanded is zero.

Consumers are more sensitive to the prices of other goods, especially for goods they do not need and/or for which there are many substitutes. When quantity demanded is sensitive to prices we say the demand is elastic. While food may be a necessity, Slim Jims are not. While there is no substitute for food (without it you die) there are substitutes for Slim Jims. Although Slim Jims is certainly the leading jerky-like snack, there are many other types of beef jerky I would switch to if the price of Slim Jims went too high.

The same can be said for any food item. While we need food to live, making demand inelastic and insensitive to prices, we do not need Doritos to live. Thus, while my food consumption may not be sensitive to changes in the price of food, my consumption of Doritos will be. Gasoline has an inelastic demand, in the presence of higher prices we curtail consumption only modestly. However, we are sensitive to changes in the price of BP gasoline, because we could also buy gasoline from Texaco. Although the demand for gasoline is inelastic, the demand for GP gasoline is elastic.

The key concept is that more choices = more sensitivity to prices = greater demand elasticity. An extreme example is perfectly elastic demand. This is a case where consumers will purchase any amount of a good but only at one particular price. Sounds weird doesn’t it? Consider a single wheat farmer, who produces only a very tiny portion of the worlds meat. The farmer is a price-taker, meaning she is such a small part of the market that she cannot negotiate a higher price, and can sell all she grows at the going price. The demand curve for this farmers wheat would look like
Perfectly Elastic Demand
Elasticity = -infinity = -\infty

She cannot sell her wheat for a penny higher than the going price, because there are many other who will sell at $4.00 / bushel.

Being a small part of the total wheat market, this farmer can sell all her production at the going price of $4.00 / bushel.

To reiterate, the more substitutes a good has, the more sensitive consumers will be to price changes, and the higher the own-price elasticity of demand will be (in absolute value). Goods that are necessities and/or have no close substitutes will have a low own-price elasticity (in absolute value). Economists have placed ranges on elasticities to designate whether they are inelastic or elastic. These ranges are shown below.

FIGURE 6.E

Own-Price Elasticity of Demand = \( E_D \)

- If \( |E_D| = 0 \) perfectly inelastic
- If \( |E_D| < 1 \) inelastic
- If \( |E_D| = 1 \) unit elastic
- If \( |E_D| > 1 \) elastic
- If \( |E_D| > \infty \) perfectly elastic
6.1) The Second Law of Demand

The first law of demand states that as the price of a good increases, quantity purchased by consumers (quantity demanded) decreases. We just covered elasticities which measure how sensitive quantity demanded is to price. Now, we want to make a stronger statement about elasticities, that should upon reflection make sense.

In the 1970's, the price of coffee skyrocketed. By then, Americans had grown accustomed to coffee and would pay outrageous prices for that morning jolt. Prices rose much, but quantity demanded decreased only a little at first. The own-price elasticity of demand was quite low. But as time went by, people started making weaker coffee. They got used to weaker coffee, and even began to prefer it. Plus, over time people discovered more substitutes for coffee, and coffee consumption declined more. In response to the price increase, quantity demanded fell just a little at first, but by much more as time went by.

Much of the Second Law of Demand has to do with preplanned expenditures. If the price of jet skis drop, I may want to buy one, but because of payments I must make on my other toys I do not have the money. Later, when those other toys are paid off, I do have the extra cash for jet skis and I make the purchase.

I was one of the last people to get a cell phone. As cell phones got cheaper, more and more of my friends bought them, but I restrained. Over time, I started noticing how convenient they are, and eventually bought one. My own-price demand elasticity of demand for cell phones went up—all due to time.

Once the production of antibiotics became cheaper (50's-60's) they were mainly used to treat human sickness. As time went on, people learned you could routinely feed antibiotics to livestock and they would grow faster. Today, over $600 million in antibiotics are sold for this purpose. Given time, people find more ways to use products that become cheaper, and more ways to avoid products that become more expensive.

The Second Law of Demand states that the more time consumers have to respond to a price change, the greater the own-price elasticity of demand will be.

Given time, people find more ways to use products that become cheaper, and more ways to avoid products that become more expensive—more time means a higher elasticity!
6.2) Cross-Price Elasticity of Demand

In the previous section we discussed the own-price elasticity of demand, which measures how consumer purchases of a good changes in response to changes in the price of that good. A cross-price elasticity measures sensitivity in response to price of other goods.

Recall earlier we asked the question: Which is a stronger substitute for beef; pork or chicken? We were concerned with how the demand for beef changes in response to a change in the price of pork and chicken. Our demand equation for boxed beef was

\[
\begin{align*}
(86) & \\
& + (-0.2545)\text{(Real Per CWT Price of Boxed Beef in 1990 Prices)} \\
& + (0.0777)\text{(Real Per CWT Price of Wholesale Pork in 1990 Prices)} \\
& + (0.1731)\text{(Real Per CWT Price of Wholesale Chicken in 1990 Prices)} \\
& + (0.0123)\text{(Real Disposable Income in 1990 Prices)} \\
& + (-0.0002)\text{(US Population in Thousands)} \\
= & \text{Quantity Demanded of Boxed Beef in Million Pounds Per Month}
\end{align*}
\]

We concluded that since a dollar increase in chicken increases beef demand more than a dollar increase in pork, chicken is a stronger substitute. Recall the \textit{ceteris paribus} assumptions used to draw the boxed beef demand curve are:

\textbf{FIGURE 6.F (FIGURE 6.B RECREATED) ESTIMATED BOXED BEEF DEMAND}

\begin{itemize}
  \item Demand Curve Assumes:
  \begin{itemize}
    \item Price Pork = 101
    \item Price Chicken = 51
    \item Income = 4,489
    \item Pop = 258,861
  \end{itemize}
\end{itemize}

Is a dollar increase pork really comparable to a dollar increase in chicken? That would be a 0.99% increase in pork prices, but a 1.96% increase in the price of chicken. To some, that is not a fair comparison. What is fair is to see how demand changes in response to the same percentage increase in price. That is what cross-price elasticities are for.
Now, let's take our boxed beef demand equation and calculate cross-price elasticities for beef with respect to pork and chicken, assuming the price of pork and chicken increase by 1%. Assume all other variables are at the values shown in Figure 6.B. Let's also assume the price of beef is $100 / cwt. Note that before either price change the quantity demanded of beef is

\[
80.6683 = \text{Quantity Demanded of Boxed Beef in Million Pounds Per Month (QD)}
\]

Now, let us see what happens when pork and chicken prices rise by 1%.

<table>
<thead>
<tr>
<th>1% Increase in Pork Prices</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>New pork price</strong> = (1.01) old price</td>
</tr>
<tr>
<td>(86)</td>
</tr>
<tr>
<td>+ (-0.2545)(100) &quot;beef price&quot;</td>
</tr>
<tr>
<td>+ (0.0777)(101*1.01 = 102.01) &quot;pork price&quot;</td>
</tr>
<tr>
<td>+ (0.1731)(51) &quot;chicken price&quot;</td>
</tr>
<tr>
<td>+ (0.0123)(4,489) &quot;income&quot;</td>
</tr>
<tr>
<td>+ (-0.0002)(258,861) &quot;pop&quot;</td>
</tr>
<tr>
<td>= 80.7468</td>
</tr>
</tbody>
</table>

A (80.7468 - 80.6683)/(80.6683) = 0.0973% Increase

<table>
<thead>
<tr>
<th>1% Increase in Chicken Prices</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>New chicken price</strong> = (1.01) old price</td>
</tr>
<tr>
<td>(86)</td>
</tr>
<tr>
<td>+ (-0.2545)(100) &quot;beef price&quot;</td>
</tr>
<tr>
<td>+ (0.0777)(101) &quot;pork price&quot;</td>
</tr>
<tr>
<td>+ (0.1731)(51*1.01 = 51.51) &quot;chicken price&quot;</td>
</tr>
<tr>
<td>+ (0.0123)(4,489) &quot;income&quot;</td>
</tr>
<tr>
<td>+ (-0.0002)(258,861) &quot;pop&quot;</td>
</tr>
<tr>
<td>= 80.7566</td>
</tr>
</tbody>
</table>

A (80.7566 - 80.6683)/(80.6683) = 0.1095% Increase
If we calculate the cross-price elasticities we find an elasticity of 0.0973 for pork and 0.1095 for chicken. The elasticity is higher with respect to chicken, so we would conclude chicken is a stronger substitute for beef than pork.

Cross-Price Elasticity of Beef With Respect to Pork:

\[ E_{ij} = \frac{\% \Delta QD_i}{\% \Delta P_j} \]

\[ = \frac{0.0973}{1} = 0.0973 \]

Cross-Price Elasticity of Beef With Respect to Chicken:

\[ E_{ij} = \frac{\% \Delta QD_i}{\% \Delta P_j} \]

\[ = \frac{0.1095}{1} = 0.1095 \]

Cross-price elasticity is higher with respect to chicken than pork—chicken is a stronger substitute for beef.

Cross-price elasticities are interpreted as the percent increase in quantity demanded due to a one percent increase in the price of another good.

Practice Question:

The pork industry expects beef prices to rise by 10% due to a lower cattle production. If the cross-price elasticity of pork with respect to beef prices is 0.015, by how much will the quantity of pork consumed rise?

Answer

The elasticity formula tells us \( E_{ij} = \frac{\% \Delta QD_i}{\% \Delta P_j} \). From the question, we know \( E_{ij} = 0.015 \) and \( \% \Delta P_j = 10\% \), then \( \% \Delta QD_i = (E_{ij} = 0.015)(\% \Delta P_j = 10\%) = 0.15 \). Quantity demanded of pork will rise 0.15% as people substitute pork for beef in response to higher beef prices.

Notice that we can tell whether a good is a substitute or complement just by the sign of its cross-price elasticity. If negative, the goods are complements, and if positive they are complements.

If complements, a rise in the price of one good causes quantity of the other good to fall, and vice versa.

If the numerator is positive, the denominator is negative, and vice versa.

Either both numerator and denominator are positive, or both are negative.

If substitutes, a rise in the price of one good causes quantity of the other good to rise, and vice versa.

Either both numerator and denominator are positive, or both are negative.
6.3) Income Elasticity of Demand

Finally, our last elasticity. We are often concerned with how demand changes in response to income. As with prices, it is real income we are interested in. After World War I, inflation rates were as high as 5,000% in Germany (compared to around 3-5% in America today). Prices were literally rising every hour. Because prices were higher, those selling goods received more money for those goods, and those paid wages demanded higher wages to meet the rising prices. In short, everyone's nominal income was rising by about 5,000%.

But people were not becoming wealthier. The rise in nominal incomes equaled the rise in nominal prices, so that people could not purchase any more goods and services than they did before. Thus, we use real income to explain demand. While the German's nominal incomes were skyrocketing, their real incomes were not. One can convert nominal income to real income exactly as one converts nominal prices to real prices.

As you probably suspect from previous sections, the income elasticity of demand is the percent change in quantity demanded divided by the percent change in real income.

**FIGURE 6.H**

INCOME ELASTICITY OF DEMAND FORMULA

\[
E_{i,j} = \frac{\text{% Change in Quantity Demanded}}{\text{% Change in Real Income}} = \frac{\text{New Quantity Demanded} - \text{Old Quantity Demanded}}{\text{Old Quantity Demanded}} = \frac{\text{New Real Income} - \text{Old Real Income}}{\text{Old Real Income}}
\]

\[
E_{i,j} = \frac{\% \Delta QD}{\% \Delta I}
\]

As people's real income rises, they tend to change their eating habits. Do they tend to eat more meat, fats, sweets, or cereals? This may be important to food manufacturers wondering how consumer's eating habits will change in the future, as the US becomes increasingly wealthy. An article by Kastens and Brester (American Journal of Agricultural Economics; May 1996) provides income elasticities for various food groups. These authors took data similar to the beef data outlined earlier, and estimated a demand equation just like we did for boxed beef. They then used this equation to calculate the income elasticity of demand, with the resultings elasticities shown below.
TABLE 6.B
INCOME ELASTICITIES FOR FOOD GROUPS

<table>
<thead>
<tr>
<th>Food Group</th>
<th>Income Elasticity of Demand</th>
<th>Normal or Inferior Good?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Meats</td>
<td>0.16</td>
<td>Normal</td>
</tr>
<tr>
<td>Eggs</td>
<td>0.02</td>
<td>Normal</td>
</tr>
<tr>
<td>Dairy</td>
<td>-0.06</td>
<td>Inferior</td>
</tr>
<tr>
<td>Fats</td>
<td>0.01</td>
<td>Normal</td>
</tr>
<tr>
<td>Cereals</td>
<td>-0.36</td>
<td>Inferior</td>
</tr>
<tr>
<td>Sweets</td>
<td>-0.24</td>
<td>Inferior</td>
</tr>
</tbody>
</table>

Source: Recreated from Kastens and Brester. The quantity variable is lbs consumed per person, and the price is price per lb.

By looking at the elasticities we can tell which goods are normal and which are inferior. As real income rises, the elasticities tell us they consume less of dairy, cereals, and sweets, making them inferior goods. Consumption of fats, eggs, and meats rises with real income, so we would conclude these are normal goods.

Real incomes in the US tend to rise about 3% per year. What does this say for the demand for dairy products? Suppose five years goes by and is associated with a 15% increase in real incomes. We could predict that the demand for dairy products will fall by:

\[ E_{i,d} = \frac{\% \Delta QD}{\% \Delta I} \]

\[ -0.06 = \frac{\% \Delta QD}{15\%} \]

which implies that \( \% \Delta QD = (15\%)(-0.06) = 0.9\% \). The dairy industry can expect for demand for dairy products to only decrease by 0.9%, which to me does not seem too large.

At this point, I should caution you about placing too much confidence in elasticity measurements. They are not perfect measurements for many reasons, and all of these reasons are statistical and beyond this course, so I will describe the problems with elasticities without proving why they are problems.

In the previous example we asked what would happen to the demand for dairy products if real incomes rise 15%. We should really not make such large extrapolations, because elasticities are only accurate for small changes in prices and income.
What if we asked what would happen to the demand for dairy products after 100 years when real incomes may increase by 300%. Our income elasticity would suggest the quantity demanded of dairy products would fall by (300%)(-0.06) = 18%. This is a very large change, and many would content unlikely. Statisticians would say this 18% is a very inaccurate number, and little weight should be placed on it. They would be more confident in predicting the outcome of a smaller change, like a 1% change of (1%)(0.06) = 0.06%.

Second, these elasticities are measured with a margin of error. Sometimes they are overestimated and sometimes they are underestimated. You know whenever the media reports the results of a poll they always say “the poll has a margin of error of 3%?” Suppose the poll is for the percent of people favoring war against Iraq, and the percentage was 52%. The 3% margin of error means the true percentage may be 49% or 55%.

Elasticity measurements also have a margin of error. An elasticity close to zero (like the income elasticity for eggs of 0.02) may actually be zero, or even negative. In the section on own-price elasticities, we found the own-price elasticity for boxed beef was −0.4. Compared to 0.02, this is large and we can be more confident that it really is negative and inelastic (recall a good has an inelastic demand if the absolute value of the own-price elasticity is greater than zero but less than one). However, if it equaled −0.9, we would have to wonder, given the margin of error, if it were really unit elastic or elastic (whether the true elasticity is actually equal to or greater than one).

If a measured elasticity is positive but close to zero, due to margins of error, it may actually be zero or negative.

If an own-price elasticity is 0.99, it is close enough to 1 that we do not truly know whether the demand is inelastic, unit elastic, or elastic.

In Table 6B the income elasticity of demand for fats was 0.01. This is so close to zero we cannot really claim to know anything about this elasticity, other than it is close to zero.

On the other hand, the income elasticity for cereals was −0.36. This is relatively large, so we can be more confident that cereals are an inferior good than we can be that fats are a normal good.
6.4) Engel's Law

We know that as incomes rise people spend more on food. Many studies, including my own, have shown that food is a normal good. But how does the percent of total income spent on food change with income? Engel's Law says it declines. Like the First Law of Demand, the validity of Engel's Law has been proven in numerous studies.

See Figure 6.1 below where US income and the percent of income spent on food is graphed for the years 1929 to 1999.

![FIGURE 6.1
Income and Food Expenditures in US](image)

Source: Household food expenditure dataset available at USDA Economic Research Service. Food expenditures include food at home and away from home.

The data clearly show that as our incomes rise, we spend a smaller portion of total income on food. Figure 6.1 demonstrates Engel's Law across time, but the law holds across space as well. Table 6.C shows the food expenditures as a percent of income for select countries. The countries are ordered from most wealthy to least wealthy, and the trend is clear. Countries with a higher income spend a smaller portion of total wealth on food.
<table>
<thead>
<tr>
<th>Country</th>
<th>Food Expenditures As Percent of Income</th>
<th>Country</th>
<th>Food Expenditures As Percent of Income</th>
</tr>
</thead>
<tbody>
<tr>
<td>United States</td>
<td>7.2</td>
<td>France</td>
<td>17.7</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>10.2</td>
<td>Israel</td>
<td>18.1</td>
</tr>
<tr>
<td>Canada</td>
<td>10.4</td>
<td>Italy</td>
<td>19.2</td>
</tr>
<tr>
<td>Netherlands</td>
<td>10.5</td>
<td>Norway</td>
<td>19.5</td>
</tr>
<tr>
<td>New Zealand</td>
<td>10.9</td>
<td>Spain</td>
<td>20.5</td>
</tr>
<tr>
<td>Singapore</td>
<td>11.3</td>
<td>Thailand</td>
<td>21.0</td>
</tr>
<tr>
<td>Austria</td>
<td>12.6</td>
<td>Portugal</td>
<td>22.1</td>
</tr>
<tr>
<td>Finland</td>
<td>12.7</td>
<td>Cyprus</td>
<td>23.9</td>
</tr>
<tr>
<td>Belgium</td>
<td>12.8</td>
<td>Mexico</td>
<td>24.0</td>
</tr>
<tr>
<td>Sweden</td>
<td>12.9</td>
<td>South Africa</td>
<td>25.0</td>
</tr>
<tr>
<td>Hong Kong</td>
<td>13.2</td>
<td>Malta</td>
<td>25.1</td>
</tr>
<tr>
<td>Denmark</td>
<td>14.1</td>
<td>Korea, Republic of</td>
<td>26.4</td>
</tr>
<tr>
<td>Ireland</td>
<td>14.5</td>
<td>Greece</td>
<td>30.1</td>
</tr>
<tr>
<td>Australia</td>
<td>14.9</td>
<td>Bolivia</td>
<td>32.2</td>
</tr>
<tr>
<td>Iceland</td>
<td>15.8</td>
<td>Poland</td>
<td>35.7</td>
</tr>
<tr>
<td>Japan</td>
<td>15.9</td>
<td>Venezuela</td>
<td>37.5</td>
</tr>
<tr>
<td>Puerto Rico</td>
<td>16.2</td>
<td>Sri Lanka</td>
<td>44.8</td>
</tr>
<tr>
<td>Switzerland</td>
<td>17.0</td>
<td>India</td>
<td>48.4</td>
</tr>
<tr>
<td>Germany</td>
<td>17.1</td>
<td>Philippines</td>
<td>52.9</td>
</tr>
</tbody>
</table>

Source: Household food expenditure dataset available at USDA Economic Research Service. Food expenditures include food at home and away from home.

Engel's Law is often demonstrated by an Engel Curve, which is a graph with income on the x-axis and food expenditures as a percent of income on the y-axis. According to Engel's Law, this graph should have a downward sloping curve.
6.5) The Mathematics of Elasticities

6.5.A) Arc Elasticity Formula

Applied economists make great use of elasticities. Elasticities are usually used to predict how economic variables (like quantity demanded) will change in response to the economic environment. Elasticities are used for computation. The purpose of this chapter is to describe several mathematical aspects of elasticities that will make using elasticities easier.

An elasticity describes the percent change in variable X with respect to the percent change in variable Y. For instance, the income elasticity describes the percent change in quantity demanded of a good (X) with respect to real income (Y). The elasticity formula is

\[ E = \frac{\text{Percent Change in } X}{\text{Percent Change in } Y} = \frac{\% \Delta X}{\% \Delta Y} \]

or

\[ E = \frac{(\text{New } X - \text{Old } X)}{(\text{New } Y - \text{Old } Y)} = \frac{\Delta X}{\Delta Y} \cdot \frac{Y}{X} \]

(1)

The formula \( E = \left(\frac{\Delta X}{\Delta Y}\right) \left(\frac{Y}{X}\right) \) is often easier to work with.

In Section 6.4 we calculated the own-price elasticity of demand for beef, who's demand curve is shown to the right. This graph shows that when price falls from 120 to 100, quantity demanded rises from 72 to 77 million lbs. We then calculated the elasticity as

\[ E_D = \frac{\Delta QD}{\Delta P} \left(\frac{P}{QD}\right) = \frac{5}{-20} \left(\frac{120}{72}\right). \]

(2)

\[ = -0.42 \]

However, what if instead of saying price decreased from 120 to 100, we said it increased from 100 to 120. The elasticity would then be calculated as

**FIGURE 6.J**
(Figure 6.B Recreated)
We see that the elasticity depends on where we started from. Did price start at 120 and decrease
to 100, or did it start at 100 and increase to 120? Economists do not like the fact that elasticities
depend on where you start, they often suggest use an alternative formula for elasticities called the
arc elasticity. Instead of having to decide whether price started at $P_1$ or $P_2$, the arc elasticity
assumes it starts at the average of $P_1$ and $P_2$. It also assumes it starts at the average of $QD_1$ and
$QD_2$. Note that the term \( \frac{\Delta QD}{\Delta P} \) will always be the same, regardless of where you start.

\[
\text{Arc } E_D = \left( \frac{\Delta QD}{\Delta P} \right) \left( \frac{P_1 + P_2}{QD_1 + QD_2} \right) = \left( \frac{\Delta QD}{\Delta P} \right) \left( \frac{P_1 + P_2}{QD_1 + QD_2} \right)
\]

In our beef example, the arc elasticity is

\[
E_D = \left( \frac{\Delta QD}{\Delta P} \right) \left( \frac{P}{QD} \right) = \left( \frac{-5}{20} \right) = -0.25 \left( \frac{100 + 120}{77 + 72} \right) = -0.37.
\]

Notice this is similar to using the average of the two elasticities of \(-0.42\) and \(-0.33\).

We also use arc elasticities for the other elasticities, such as income elasticity. The formula is the
same except for, instead of using the term \((P_1+P_2)\), you would use \((\text{Income}_1+\text{Income}_2)\).
6.5.B) Elasticities and Derivatives

Recall from the previous section we said elasticities could be written as  \( E = \left( \frac{\Delta X}{\Delta Y} \right) \left( \frac{Y}{X} \right) \). If the change in \( Y \) is small and you decide to use calculus, this formulation is especially convenient.

Recall that the derivative \( \frac{dX}{dY} \) equals the ratio \( \left( \frac{\Delta X}{\Delta Y} \right) \) as the value of \( \Delta Y \) becomes infinitely small.

This implies that if \( X \) is a function of \( Y \), i.e. \( X = f(Y) \), and you know that function, elasticities can be written as

\[
E = \left( \frac{dX}{dY} \right) \left( \frac{Y}{X} \right) = \left( \frac{df(Y)}{dY} \right) \left( \frac{Y}{X} \right) = f'(Y) \left( \frac{Y}{X} \right).
\]

Sometimes we know the function \( f(Y) \). Other times we estimate it like our beef demand equation. Recall in Section 6.3 we estimated the following equation for beef demand

(3) Quantity Demanded of Boxed Beef in Million Pounds = \((86) + (-0.2545)(P_{Beef}) + (0.0777)(P_{Pork}) + (0.1731)(P_{Chicken}) + (0.0123)(Income) + (-0.0002)(Population)\).

Let us rewrite this as

(4) \( QD_{Beef} = (86) + (-0.2545)(P_{Beef}) + (0.0777)(P_{Pork}) + (0.1731)(P_{Chicken}) + (0.0123)(I) + (-0.0002)(N) \).

Notice that since the derivative of \( QD_{Beef} \) with respect to \( P_{Beef} \) is \(-0.2545\), we can calculate the own-price elasticity of demand as

\[
E_D = \left( \frac{\partial QD_{Beef}}{\partial P_{Beef}} \right) \left( \frac{P_{Beef}}{QD_{Beef}} \right) = (-0.2545) \left( \frac{P_{Beef}}{QD_{Beef}} \right).
\]

As mentioned in the previous section, this elasticity will depend on where you are on the demand curve. The Table below shows the elasticity value for various points on the demand curve:

<table>
<thead>
<tr>
<th>Quantity Demanded</th>
<th>Price</th>
<th>Own-Price Elasticity</th>
</tr>
</thead>
<tbody>
<tr>
<td>70</td>
<td>130</td>
<td>-0.40</td>
</tr>
<tr>
<td>75</td>
<td>110</td>
<td>-0.37</td>
</tr>
<tr>
<td>80</td>
<td>90</td>
<td>-0.29</td>
</tr>
<tr>
<td>85</td>
<td>70</td>
<td>-0.21</td>
</tr>
</tbody>
</table>
Notice as we move down the demand curve (lower prices and higher quantities) the elasticity gets smaller. This is always true with linear demand curve. If look in (5), you will see that the derivative is constant (because demand is linear) but as the price rises, quantity falls, making the term \( \left( \frac{P_{\text{Beef}}}{Q_{\text{D Beef}}} \right) \), and hence the elasticity, larger in absolute value.

Derivatives can be used for any type of elasticity. Take the income elasticity for example, if quantity demanded is a function of income \( I \), the income elasticity is

\[
\text{(6)} \quad E_I = \left( \frac{\partial QD}{\partial I} \right) \left( \frac{1}{QD} \right).
\]

Using the demand equation in (3), if real income is 4,489 and the current quantity demanded is 80 million lbs, the income elasticity is

\[
\text{(7)} \quad E_I = \left( \frac{4,489}{80} \right) = 0.69.
\]